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EARNED VALUE ANALYSIS APPROACH BY EXTENDING NEW KEY PERFORMANCE INDICATORS IN A NEW FRAMEWORK UNDER A GREY ENVIRONMENT

Abstract. *An inseparable part of project management is the existence of uncertainty. There are various methodologies and tools to handle the impact of uncertainty on project performance.Todenote the uncertainty, we use grey theory because ofits accuracy and convenience in modeling uncertainty.In this paper, a new grey-based earned value technique with a new extended set of key performance indicators (KPIs)is proposed to measure the progress, performance, cost and time estimate at completion of the construction projects under uncertainty circumstances. A new formulation of cost and time estimation at completion is introduced that is based on using a newly developed weighting method called W-TODIM. The proposed model improves managing different aspects of a project through a flexible and intelligent approach. Finally, a case study of construction project from the literature is presented to show the efficiency of presented model.*

Keywords:*Uncertainty, Earned value management, Key performance indicators(KPIs), Grey theory, Construction projects*

JEL Classification: C02, C44, D81, L74, M10.

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1. Introduction

Always, construction projectsare faced with many risks to achieve predetermined objectives over their life cycle.Dynamism, complexities and specific nature of construction projects are quickly deteriorated andrisks could behazardousfor project components if not properly reduced or removed(Taillandieret al., 2015;Rostami et al., 2015).Risks come from uncertainties that exist in all projects.Project risk definition could be a vague status or an event which has a positive or negative effect on goals of a project (PMI, 2013).Since uncertainty is one of the risk factors, the term uncertainty should be described and explained.

Due to increasing costs and complexity in the projects, as well as growing uncertainty and existing risks, project managers employ risk management techniques in the planning and control of projects to reduce the risk and diversion of the project from the predesignated goals.The range of uncertainty in projects is significant and many of the project management activities, focus on defining and deciding on a set of possible measures against project’s uncertainties from the early stages of the project life cycle. Part of the uncertainty in projects, refer topossibility to change the project performance criteria, such as cost, time and quality.Uncertainties can also be found on issues, such as ambiguity and unpredictability in understanding project teamwork and stakeholder’s behavior, lack of information, lack of an explicit structure to consider project issues,known and unknown resources of deviations in the project and other related issues(Banihashemi et al., 2017; Anantatmula and Fan, 2018).

Two types of uncertainties can be categorized inreal-world problems that are random uncertainty and perceptual uncertainty. To describe and study the aspect of the second type, grey theory has been developed in situations with low data or incomplete qualitative information. Grey theory works well in fuzzy conditions (Deng, 1989). Grey systems have been named based on thecolor of subjects under investigation. So that the colors brightness rate represents the clarity level of information and data. Accordingly, systems with well-defined information are white systems, systems with ill-defined data are black systems and systems with somewhat known and partly unknown information are called grey systems (Li and Liu, 2008; Li et al., 2014).

Lin et al.(2004) looked at the state-of-the-art of the grey system theory and its applications since it’s creation. Then, its several successful applications were mentioned by examining the history of this theory.Grey mathematical

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programming models were employed for project time-cost-quality trade-offs with uncertain conditions by RazaviHajiaghaet al. (2015).Rahimnia et al.(2011) applied a methodology to evaluate the qualitative quiddity of organizational attitudes. As an aim of their study, vision of several universities was assessed and ranked by the grey theory.Slavek and Jović(2012) employed grey theory to anticipate and specify software projects ranking by their success and then evaluated the quality of them.Bai and Sarkis(2013) presented a method for evaluating critical success factors (CSFs) for implementing business process management (BPM), using a new gray-based decision-making approach. It can efficiency use uncertainty and indistinctive problems in order to help project managers making appropriate investment decisions for the BPM.Bhattacharyya (2015) presented a model based on multiple attributes decision making(MADM) methodology to select project portfolio for research and development(R&D). Firstly, the study discussed the uncertainty which exists on the preferences of decision-makers about alternatives or attributes of the project. Secondly as a coping with the uncertainty, the grey theory is applied to the project ranking.Prioritizing entrepreneurship main risks in non-profit financial funds based on TODIM method under grey environment in Iran was carried out by Ekhtiari et al. (2016).

Monitoring of project performance information has become an extremely critical taskdue to complexity of the activities and conditions affecting the project. In this paper, key performance indicators (KPIs) are employed to evaluate the power of project success in EVM.EVMassists the project manager to handleproject from two perspectives. The first one is to acknowledgeexisting performance indicators, and the second one is to present future prediction (Noori et al., 2008).Thus, by reviewing the research, it is noticed that in the project control and management, the indicators and estimates, weights of the indices from the MADMperspectivesand the uncertainty with consideration of the grey theory in the analysis of the earned value management (EVM) are not considered. Therefore, in this paper, taking into account the assumptions mentioned, a new attempt is made to analyze the earned value, which is closer to real life conditions of construction projects.

In summary, the main features ofthis paper that separate it from the similar studies in thisfield are as follows:

- Grey theory is presented and applied inproject performance monitoring and analyzing. It provides the decision maker (DM) with more flexibility in expressing uncertainty. Furthermore, the project performance calculations would bemore flexible, simple, and convenience.

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- A new set of KPIs is presented to handle and evaluate a construction project.Moreover,these KPIs are separated into two groups of project implementation and risk indicators.For this purpose, five KPIs of critical aspectsare coupled with additional four KPIs of risk.It should be mentioned that KPIs of risk constitute different risk indicators for cost and time.
- Weight of each KPI in grey-based estimation equations is computed by developing new concept of iterative multi-criteria analysis method called TODIM (an acronym in Portuguese for interative multi-criteria decision making). In other words, the importance of KPIs is both denoted by direct judgement of the DMs and by computing weights based on the gatheredjudgements.

To conceive comprehension, the rest of this paper is divided into 4 sections. A brief overview on grey numbers and grey-based EVM method are given in section 2. The proposed analytical approach is introduced in section 3. Then, a case study from the literature is presented and solved in section 4. Discussion of result is represented in section5. Finally, the paper is concluded in the last section.

2.Presented greyEVM calculations

The EVM is a powerful tool that permits practitioners, project managers, and others to monitorthe project condition during the life cycle of project. It is possible to gain more efficiency in managing the handle of project, programs, and portfolios. The opportunities are measuredobviously by EVM to keep monitoring over important issues, such as budget, scope, and schedule of different types of projects (Salari etal., 2014;Simion and Marin, 2018).

Measuring EV is not simple and how to determine it is a matter of discussion between EVM practitioners.The basic concept of EVM is to compare the amount of earned value against the money paid for each activity. Firstly, each activity progress is determined by expert opinion and they have to be transformed into the grey progress percent by means of linguistic terms showed in Table1.

$$\otimes\, GPP_i = \left[\underline{a_i}\,.\,\overline{a_i}\right] \tag{1}$$

Secondly, a formal definition of grey-based earned value for activity i can be obtaining as follows:

$$\otimes\, EV_i = \otimes\, GPP_i * BAC_i = \left[\underline{E_i}.\overline{E_i}\right] = [a_i * BAC_i.\, a_i * BAC_i] \tag{2}$$

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where i=1,2,…,n is the total number of project activities and BAC_i denots the Budget of Completion.

The overall grey-based earned value could be calculated by rolling up all the $\otimes EV_i$ for activity as follows:

$$\otimes EV_O = \sum_{i=1}^{n} \otimes EV_i = \left[\sum_{i=1}^{n} \underline{E_i} . \sum_{i=1}^{n} \overline{E_i}\right] = \left[\underline{E_1} . \overline{E_2}\right] \tag{3}$$

Table 1. Dedicated linguistic terms associated with grey numbers

Linguistic terms	Grey numbers
Near to the beginning	[0,0.1]
Very low	[0.1,0.2]
Low	[0.2,0.3]
Less than half	[0.3,0.4]
Half	[0.4,0.5]
More than half	[0.5,0.6]
High	[0.6,0.7]
Very high	[0.7,0.8]
Near to the end	[0.8,1]

Subsequently, according to PMI (2013), cost performance index(CPI), schedule performance index (SPI), schedule cost index (SCI) and cost etimation at completion (EAC) grey-based calculations are presented using Eqs.(4)-(8):

$$\otimes SPI = {}^{\otimes EV_O}/_{PV} = \left[\underline{{}^{E_1}/_{PV}} . \overline{{}^{E_2}/_{PV}}\right] \tag{4}$$

$$\otimes CPI = {}^{\otimes EV_O}/_{AC} = \left[\underline{{}^{E_1}/_{AC}} . \overline{{}^{E_2}/_{AC}}\right] \tag{5}$$

$$\otimes SCI = \otimes CPI \times \otimes SPI = \begin{bmatrix} min\left(\underline{CPI} \times \underline{SPI} . \underline{CPI} \times \overline{SPI} . \overline{CPI} \times \underline{SPI} . \overline{CPI} \times \overline{SPI}\right) . \\ max\left(\underline{CPI} \times \underline{SPI} . \underline{CPI} \times \overline{SPI} . \overline{CPI} \times \underline{SPI} . \overline{CPI} \times \overline{SPI}\right) \end{bmatrix}$$

$$= \begin{bmatrix} min\left(\underline{{}^{E_1}/_{AC}} \times \underline{{}^{E_1}/_{PV}} . \underline{{}^{E_1}/_{AC}} \times \overline{{}^{E_2}/_{PV}} . \overline{{}^{E_2}/_{AC}} \times \underline{{}^{E_1}/_{PV}} . \overline{{}^{E_2}/_{AC}} \times \overline{{}^{E_2}/_{PV}}\right) . \\ max\left(\underline{{}^{E_1}/_{AC}} \times \underline{{}^{E_1}/_{PV}} . \underline{{}^{E_1}/_{AC}} \times \overline{{}^{E_2}/_{PV}} . \overline{{}^{E_2}/_{AC}} \times \underline{{}^{E_1}/_{PV}} . \overline{{}^{E_2}/_{AC}} \times \overline{{}^{E_2}/_{PV}}\right) \end{bmatrix} = \left[\underline{SC} . \overline{SC}\right] \tag{6}$$

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$$\otimes EAC = \frac{BAC}{\otimes CPI} = [BAC.BAC] \times \left[\frac{1}{\underline{CPI}}.\frac{1}{\overline{\overline{CPI}}}\right] = [\underline{eac}.\overline{eac}] \quad (7)$$

$$\otimes EAC = AC + \left[\frac{BAC - \otimes EV_O}{\otimes SCI}\right]$$
$$= AC + [\min\begin{pmatrix}(BAC - E_2) \times \frac{1}{\underline{SC}}.(BAC - E_2) \times \frac{1}{\overline{\overline{SC}}}. \\ (BAC - E_1) \times \frac{1}{\underline{SC}}.\ (BAC - E_1) \times \frac{1}{\overline{\overline{SC}}}\end{pmatrix}. \quad (8)$$
$$\max\begin{pmatrix}(BAC - E_2) \times \frac{1}{\underline{SC}}.(BAC - E_2) \times \frac{1}{\overline{\overline{SC}}}. \\ (BAC - E_1) \times \frac{1}{\underline{SC}}.\ (BAC - E_1) \times \frac{1}{\overline{\overline{SC}}}\end{pmatrix}]$$

where PV and AC are planned value and actual cost, respectively.

Due to lack of attention to time units in EVM calculations, three methods have been introduced to evaluate the schedule performance. Lipke(2003) proposed the earned schedule (ES)totranslate the EV of a given condition date interms of time units. In other words, principles for ES are exactly similar to earned value. New formula forthe ES under the grey environment is determined as below:

$$ES = t + \left[\frac{EV_i - PV_t}{PV_{t+1} - PV_t}\right] \text{ i=1,2}$$
$$\otimes ES = [ES_1.ES_2] \quad (9)$$

where t is denoted as a value so that $PV_t < EV < PV_{t+1}$.In other words,t represents the closest period which the current EV is more than the planned value of that period.

With ES, a different index for project schedule performance can be calculated which is called $SPI_{(t)}$:

$$\otimes SPI_{(t)} = \frac{\otimes ES}{AD} = \left[\underline{{ES_1}/_{AD}}.\overline{{ES_2}/_{AD}}\right] \quad (10)$$

General formula for time estimation at completion is represented by Eq. (11) which performance factor (PF) denoted by completion trend of remained activities

$$\otimes EAC_{(t)} = AD + \left[\frac{PD - \otimes ES}{PF}\right] \quad (11)$$

• PF = 1: The duration of the remained activities is as planned.

$$\otimes EAC_{(t)} = AD + [PD - \otimes ES] = [AD + PD - \otimes \overline{es}.AD + PD - ES_1] \quad (12)$$

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• $PF = \otimes SPI_{(t)}$:The trend of $\otimes SPI_{(t)}$ changes the activities duration which did not completed.

$$
\otimes EAC_{(t)} = AD + \left[\frac{PD - \otimes ES}{\otimes SPI_{(t)}}\right]
$$

$$
= AD + \begin{bmatrix} \min\begin{pmatrix} (PD - ES_2) \times \frac{1}{\underline{SPI_{(t)}}} . (PD - ES_2) \times \frac{1}{\overline{SPI_{(t)}}} . \\ (PD - ES_1) \times \frac{1}{\underline{SPI_{(t)}}} . (PD - ES_1) \times \frac{1}{\overline{SPI_{(t)}}} \end{pmatrix} . \\ \max\begin{pmatrix} (PD - ES_2) \times \frac{1}{\underline{SPI_{(t)}}} . (PD - ES_2) \times \frac{1}{\overline{SPI_{(t)}}} . \\ (PD - ES_1) \times \frac{1}{\underline{SPI_{(t)}}} . (PD - ES_1) \times \frac{1}{\overline{SPI_{(t)}}} \end{pmatrix} \end{bmatrix} \tag{13}
$$

• PF = $\otimes SCI_{(t)}$: The trend of $\otimes SCI_{(t)}$ changes the activities duration which didnot complete.

$$
\begin{aligned}
\otimes SCI_{(t)} &= \otimes SPI_{(t)} \times \otimes CPI \\
&= [\min\left(\underline{SPI_{(t)}} \times \underline{CPI} . \underline{SPI_{(t)}} \times \overline{CPI} . \overline{SPI_{(t)}} \times \underline{CPI} . \overline{SPI_{(t)}} \times \overline{CPI}\right) . \\
&\quad \max\left(\underline{SPI_{(t)}} \times \underline{CPI} . \underline{SPI_{(t)}} \times \overline{CPI} . \overline{SPI_{(t)}} \times \underline{CPI} . \overline{SPI_{(t)}} \times \overline{CPI}\right)] \\
&= [\min\begin{pmatrix} \underline{ES_1/AD} \times \underline{E_1/AC} \cdot \underline{ES_1/AD} \times \overline{E_2/AC} \cdot \\ \overline{ES_2/AD} \times \underline{E_1/AC} \cdot \overline{ES_2/AD} \times \overline{E_2/AC} \end{pmatrix} . \\
&\quad \max\begin{pmatrix} \underline{ES_1/AD} \times \underline{E_1/AC} \cdot \underline{ES_1/AD} \times \overline{E_2/AC} \cdot \\ \overline{ES_2/AD} \times \underline{E_1/AC} \cdot \overline{ES_2/AD} \times \overline{E_2/AC} \end{pmatrix}] = [\underline{SC_{(t)}} . \overline{SC_{(t)}}]
\end{aligned} \tag{14}
$$

Regarding to $\otimes SCI_{(t)}$ as a performance factor,the $\otimes EAC_{(t)}$ is calculated as follows:

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$$\otimes EAC_{(t)} = AD + \left[\frac{PD - \otimes ES}{\otimes SCI_{(t)}}\right] = AD + \left[\frac{(PD - ES_2 . PD - ES_1)}{\left(\frac{ES_1}{AD} . \frac{ES_2}{AD}\right)}\right]$$

$$= AD + \left(\begin{array}{l} \min\left[\frac{PD - ES_2}{\frac{ES_1}{AD}} . \frac{PD - ES_2}{\frac{ES_2}{AD}} . \frac{PD - ES_1}{\frac{ES_1}{AD}} . \frac{PD - ES_2}{\frac{ES_2}{AD}}\right] . \\ + \max\left[\frac{PD - ES_2}{\frac{ES_1}{AD}} . \frac{PD - ES_2}{\frac{ES_2}{AD}} . \frac{PD - ES_1}{\frac{ES_1}{AD}} . \frac{PD - ES_2}{\frac{ES_2}{AD}}\right] \end{array} \right) \quad (15)$$

Where AD = Actual duration, PD = Planned duration.

3. Proposed methodology

3.1. KPIs overview and calculations

As regards to the fact that the tools of this study for evaluating the project performance are based on KPIs,definition of KPIs is significant to provide a preliminary understandingof the implemented method.KPIsconcentrate on differentaspects of project performance, output,and outcomes,leading tosuccess of the project. Any project will only have KPIs in its area, if a KPI is significant to the project. Indeed, KPIs are project-based (Wiley, 2015). The process of choosing the appropriate KPIs would be difficult,whereas determining KPIs or creating an archive or set of KPIs is easy (Kerzner, 2017).

Sometimes, information and data collection methods must be extended at the same time as the project progresses, but in this sub-section, generally we introduce the following KPIs in construction projects:cost performance index (CPI), schedule performance index (SPI), quality performance index(QPI), stakeholder satisfaction performance indicator (SSPI), economical performance indicator (EPI), political performance indicator (PPI), management performance indicator (MPI), construction performance indicator (CsPI), cost risk performance indicator(CRPI), time risk performance indicator(TRPI).

Generally, significance of the indicators and their influential factors can be proved and disproved by the judgment of experts. The value of a particular KPI is determined by rolling up the normalized individual weight ofaninfluential factor(IF) multiplied by its value(F_i); where IF values and IF weightings are obtained through linguistic tables and weighting methods, respectively. It should be noted that the values of grey RPIs ($\otimes CRPI$or$\otimes TRPI$)are calculated by adding four individual group values multiplied with their respective normalized group weightings. For instance, the $\otimes QPI$ and $\otimes CRPI$ are obtained as follows:

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$$\otimes QPI = \sum_{i=1}^{n} (\otimes \alpha_{Q_i} \times \otimes F_i) \tag{16}$$

$$\otimes CRPI = (\otimes w_1 \times \otimes EPI) + (\otimes w_2 \times \otimes CoPI) + (\otimes w_3 \times \otimes PPI) + (\otimes w_4 \times \otimes MPI) + (\otimes w_5 \times \otimes CsPI) \tag{17}$$

Where $\otimes \alpha_{KPI_i}$ is internal weightings of the respective IFs, which determined by DMs;$\otimes F_i$ are values of IFs(ranging from [0, 0.]to [0.9,1]); $\otimes w_i$ $(i = 1, \ldots .5)$and $\otimes z_j$ $(j = 1. \ldots .5)$are weights of constituent KPIs in $\otimes CRPI$ and $\otimes TRPI$ calculated by means of relative importance index(RII). According to Muhwezi(2014), the RII formula is computed by followingEq.:

$$\otimes RII = \frac{\sum \otimes W}{A \times N} \tag{18}$$

$\otimes W$=Assigned weight forany factor (Table 1);
A =Highest weight (i.e. [0.9,1] in this case) and;
N = Total number of respondents.

Table 2 shows the scale of attribute weights and ratings.

Table 2. The scale of attribute weights $\otimes w$ and attribute ratings $\otimes A$

Scale	$\otimes w$	Scale	$\otimes A$
Very low (VL)	[0.0, 0.1]	Very poor (VP)	[0, 1]
Low (VL)	[0.1, 0.3]	Poor (P)	[1, 3]
Medium low (ML)	[0.3, 0.4]	Medium poor (MP)	[3, 4]
Medium (M)	[0.4, 0.5]	Fair (F)	[4, 5]
Medium high (MH)	[0.5, 0.6]	Medium good (MG)	[5, 6]
High (H)	[0.6, 0.9]	Good (G)	[6, 9]
Very high(VH)	[0.9, 1]		

3.2.$\otimes$ $GIEAC_{(\$)}$calculations

New grey improved cost estimate at completion($\otimes GIEAC_{(\$)}$) is presented as follows:

$$\otimes GIEAC_{(\$)} = AC + \frac{BAC - \otimes EV_O}{(\otimes WK_1 \times \otimes CPI + \otimes WK_2 \times \otimes SPI + \otimes WK_3 \times \otimes QPI + \otimes WK_4 \times \otimes SFPI + \otimes WK_5 \times \otimes CRPI)} \tag{19}$$

where $\widetilde{\widetilde{WK}}_i$denotes the weighting of each KPI used in the formation of$\otimes GIEAC_{(\$)}$ and is obtained by W-TODIMmethod, which is proposed in the following:

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Steps of proposed W-TODIM method: In this paper, a method of TODIM for weighting (called W-TODIM) is introduced for assigning weight to each KPI according to the DM's opinion. The recognized method named presented by Gomes and Lima (1992) has been utilized to solving MADM problems based on the Prospect theory.

As the basic step of the W-TODIM method, consider on MADM problem with a set of six key performance indicators $KPI = \{CPI.SPI.QPI.SSPI.SFPI.RPI\}$ and a set of mDMs,$DM = \{DM_1.DM_2.\cdots.DM_m\}$, whose $W = [W_1.W_2.\cdots.W_6]^T$ is the weight vector of the KPIs such that $0 \leq W_i \leq 1$ for all $i = 1.\cdots.6$.

The procedure of W-TODIM method is presented in the following steps:

Step 1. Establish DM's opinion matrix for importance of each KPI. Noted that, the values are nominal at first. Numerical matrix is then performed using Table 1. Moreover, the evaluation of$KPI_i$with respect to each DM (DM_j)is denoted by W_{ij}, $i = 1.2.\ldots.6$, $j = 1.2.\ldots.m$. For convenience, the decision matrix is denoted by $W = \left(W_{ij}\right)_{6\times m}$.

Step 2.The relative weight(subjective preference)SP_{rj} for each DM,DM_j, is calculated with respect to the reference DM (best DM in terms of science, background, and so on)SP_r as $SP_{rj} = \frac{SP_j}{SP_r}$, $j = 1.2.\ldots.m$, where $SP_r = \max_j\{SP_j\}$.

Step 3. For each key performance indicators pair KPI_u and KPI_v, calculate the perceived dominance degree of KPI_u over KPI_v with respect to DM_j by:

$$Q_{uv}^j = Q_j(KPI_u.KPI_v) = \begin{cases} \sqrt{\dfrac{(y_{uj} - y_{vj})SP_{rj}}{\sum_{j=1}^m SP_{rj}}} & if\ y_{uj} - y_{vj} > 0 \\ 0 & if\ y_{uj} - y_{vj} = 0 \\ -\dfrac{1}{\theta}\sqrt{\dfrac{(y_{uj} - y_{vj})\sum_{j=1}^m SP_{rj}}{SP_{rj}}} & if\ y_{uj} - y_{vj} < 0 \end{cases} \tag{20}$$

$u.v = 1.2.\cdots.n$, $j = 1.2.\cdots.m$,

Where $\theta > 0$ is defined as the attenuation factor of the loss and a larger θ means a smaller degree of loss aversion. Moreover, $\left(y_{uj} - y_{vj}\right) > 0$ denotes the gain of KPI_u over KPI_v with respect to DM_j, while $y_{uj} - y_{vj} < 0$ denotes the loss of KPI_u over KPI_v with respect to $DM_j.u.v = 1.2.\ldots.n, j = 1.2.\ldots.m$.

Step 4.Calculate the collective perceived dominance degree for each alternative pair KPI_u andKPI_v by:

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$$\gamma_{uv} = \sum_{j=1}^{m} Q_{uv}^{j} . \qquad u.v = 1.2.\cdots.m. \tag{21}$$

Step 5. Compute the overall performance degree of KPI_u rather than all KPIs:

$$T_u = \sum_{\substack{v=1 \\ v \neq u}}^{6} \gamma_{uv} \tag{22}$$

Step 6. Calculate the normalized weight of any KPI.

$$WK_u = \frac{T_u}{\sum_{u=1}^{6} T_u} \tag{23}$$

Table 3. Matrix of KPI's score against DM's opinion

KPI	DM					
	DM_1	DM_2	…	DM_j	…	DM_m
$KPI_1 = CPI$	W_{11}	W_{12}	…	W_{1j}	…	W_{1m}
$KPI_2 = SPI$	W_{21}	W_{22}	…	W_{2j}	…	W_{2m}
$KPI_3 = QPI$	W_{31}	W_{32}	…	W_{3j}	…	W_{3m}
$KPI_4 = SSPI$	W_{41}	W_{42}	…	W_{4j}	…	W_{4m}
$KPI_5 = SFPI$	W_{51}	W_{52}	…	W_{5j}	…	W_{5m}
$KPI_6 = CRPI$	W_{61}	W_{62}	…	W_{6j}	…	W_{6m}

3.3. $\otimes$ $GIEAC_{(t)}$ calculations

New grey improved time estimate at completion($\otimes$ $GIEAC_{(t)}$) is presented as follows:

$$PF = \otimes Z_1 \times \otimes CPI + \otimes Z_2 \times \otimes SPI_{(t)} + \otimes Z_3 \times \otimes QPI + \otimes Z_4 \times \otimes SFPI + \otimes Z_5 \times \otimes TRPI \tag{24}$$

$$\otimes GIEAC_{(t)} = AD + \frac{PD - \otimes ES}{\otimes Z_1 \times \otimes CPI + \otimes Z_2 \times \otimes SPI_{(t)} + \otimes Z_3 \times \otimes QPI + \otimes Z_4 \times \otimes SFPI + \otimes Z_5 \times \otimes TRPI} \tag{25}$$

where $\otimes Z_i$ denotes the weighting of each KPI used in the formation of $\otimes GIEAC_{(t)}$, is obtained by W-TODIM method which is proposed in Section 3.2.

3.4. Ranking

A method proposed by Liu(2016) is applied for comparison of grey numbers whichaccording to this method, the steps of the grey ranking method are presented as follows:

(1) An ideal grey number $\otimes A^*$ is determined as follows:

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$$\otimes A^{*} = \left[\max\left(\underline{A_1}.\underline{A_2}\right).max\left(\overline{A_1}.\overline{A_2}\right)\right] = \left[\underline{A^{*}}.\overline{A^{*}}\right] \tag{26}$$

(2) The crisp output of $\otimes A_1$ and $\otimes A^{*}\left(Co(\otimes A_1.\otimes A^{*})\right)$and $Co(\otimes A_2.\otimes A^{*})$ is computed as follows:

$$Co(\otimes A_1.\otimes A^{*}) = {\left(\overline{A_1} - \underline{A^{*}}\right)^2}\big/_{2} \tag{27}$$

$$Co(\otimes A_2.\otimes A^{*}) = {\left(\overline{A_2} - \underline{A^{*}}\right)^2}\big/_{2} \tag{28}$$

(3) Specify status of $\otimes A_1$ and $\otimes A_2$ relative to each other:

(i) If $Co(\otimes A_1.\otimes A^{*}) < Co(\otimes A_2.\otimes A^{*})$, then $\otimes A_1 <\otimes A_2$; (29)

(ii) If $Co(\otimes A_1.\otimes A^{*}) = Co(\otimes A_2.\otimes A^{*})$, then $\otimes A_1 =\otimes A_2$; (30)

(iii) If $Co(\otimes A_1.\otimes A^{*}) > Co(\otimes A_2.\otimes A^{*})$, then $\otimes A_1 >\otimes A_2$. (31)

3.5.Interpretation

Interpretation of the EV estimation's results is vital in order to arrive at a proper conclusion on the future status of the project in terms of cost and time. With regards to conventional procedure of EV,estimated cost and time have to be comparedwith BAC and PD, respectively.In Tables 4 and 5, comparisons and related explanation are provided.

Table 4. Explanationof $\otimes GIEAC_{(\$)}{}'s$ scenarios

Scenario	Status	Explanation
1	$Co(\otimes GIEAC_{(\$)}.\otimes A^{*}) = Co(BAC.\otimes A^{*})$	The project will be completed at a similar cost to the BAC.
2	$Co(\otimes GIEAC_{(\$)}.\otimes A^{*}) < Co(BAC.\otimes A^{*})$	The project will be completed at a lower cost than the BAC.
3	$Co(\otimes GIEAC_{(\$)}.\otimes A^{*}) > Co(BAC.\otimes A^{*})$	The project will be completed with more cost than the BAC.

Table 5. Explanation of $\otimes GIEAC_{(t)}{}'s$scenarios

Scenario	Status	Explanation
1	$Co(\otimes GIEAC_{(t)}.\otimes A^{*}) = Co(PD\ .\otimes A^{*})$	The project will be completed at a similar time to the BAC.
2	$Co(\otimes GIEAC_{(t)}.\otimes A^{*}) < Co(PD\ .\otimes A^{*})$	The project will be completed at a lower time than the PD.
3	$Co(\otimes GIEAC_{(t)}.\otimes A^{*}) > Co(PD\ .\otimes A^{*})$	The project will be completed with more time than the PD.

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4. Case study

In this section, a case study from the recent literature (Salari et al., 2014)which refers to a construction project is presented and solved to illustratetheefficiency of the proposed grey model in earned value methodology. In this case, the first level ofwork breakdown structure includes six activities,such as roofing, interior, electrical, plumbing, framing, and concrete that each of them is divided into three work packages.For simplicity, we deal only with the WBS in first and second levels, despite the fact that it didnot limited to the two levels.

This case is take long a period of 14 months that the PV and AC of the project up to month 8 are brought in Table 6. Also,linguistic terms of $\otimes\, GPP_i$ and BACfor each activity are depicted in Table 7.

Table 6. PV and AC of the project (Salari et al., 2014)

Month	1	2	3	4	5	6	7
PV	4000	8000	11500	14000	16500	19600	23200
AC	3500	7500	10100	14460	16760	19360	22460
Month	8	9	10	11	12	13	14
PV	27500	28000	31500	37000	42500	47500	49500
AC	24410						

The linguistic terms of activities are transformed into $\otimes\, GPP_i$using Table 1 and are stated in Table 7.

By regarding Eq. (2), for instance$\otimes\, EV_{12}$ is calculated as follows:

$$\otimes\, EV_{12} = \otimes\, GPP_{12} * BAC_{12} = [0.0.1] \times 1200 = [0.120] \quad (32)$$

Table 7. Information of activites (Salari et al., 2014) and their earned value

	Activity	BAC($)	Progress	$\otimes\, GPP_i$	$\otimes\, EV_i$
1	Pour foundation	8000	Completed	[1,1]	[8000,8000]
2	Install patio	3400	Very high	[0.9 ,1]	[3060,3400]
3	Stairway	2500	half	[0.4,0.5]	[1000,1250]
4	Frame exterior walls	2500	Not started	[0,0]	[0,0]
5	Frame interior walls	3000	Low	[0.1,0.3]	[300,900]
6	Install roofing trusses	1250	Not started	[0,0]	[0,0]
7	Install water lines	1900	Low	[0.1,0.3]	[190,570]
8	Install gas lines	2300	Low	[0.1,0.3]	[230,690]
9	Install bath and fixture	850	Not started	[0,0]	[0,0]
10	Install wiring	950	Very low	[0,0.1]	[0,95]
11	Install outlet/switches	1350	Not started	[0,0]	[0,0]
12	Install fixtures	1200	Very low	[0,0.1]	[0,120]

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13	Install drywalls	2400	half	[0.4,0.5]	[960,1200]
14	Install carpets	3200	Not started	[0,0]	[0,0]
15	Paintings	5600	Less than half	[0.3,0.4]	[1680,2240]
16	Install felt	3600	more than half	[0.5,0.6]	[1800,2160]
17	Install shingles	2600	Very low	[0,0.1]	[0,260]
18	Install vents	2900	Not started	[0,0]	[0,0]

Based on Table 7 and Eq. (3), the overall EV for all the activities is as below:

$$\otimes EV_O = \sum_{i=1}^{18} \otimes EV_i = [17220.20885] \tag{33}$$

The PV and AC ineighth month are \$ 27,500 and \$ 24,410, respectively, as regards to Table 6. Hence, using Eq.(4) and Eq. (5),we can obtain theSPI and CPI,respectively.

$$\otimes SPI = \left[\frac{17220}{27500} . \frac{20885}{27500}\right] = [0.6261 . 0.7594] \tag{34}$$

$$\otimes CPI = \left[\frac{17220}{24410} . \frac{20885}{24410}\right] = [0.7054 . 0.8555] \tag{35}$$

As stated in the second part of this study, the earned schedule calculations are computed in the following.The calculation of the ES is based on projecting each of the two members of grey $\otimes EV_O$on the baseline as given inTable 7. For instance, ES_i is obtained as follows:

$t_1 = 5 \; as \; PV_5 < E_1^U = 17720 < PV_6$;therefore,

$$ES_1 = N_1 + \left(\frac{E_1 - PV_5}{PV_6 - PV_5}\right) = 5 + \left(\frac{17720 - 16500}{19600 - 16500}\right) = 5.2322 \tag{36}$$

$$\otimes ES = [5.2322 \text{،} 6.3569] \tag{37}$$

The $\otimes SPI_t$ could be determined by Eq. (10):

$$\otimes SPI_t = \frac{\otimes ES}{AD} = \left[{}^{5.2322}/_{8} . {}^{6.3569}/_{8}\right] = [0.654 . 0.794] \tag{38}$$

Then, the values of other KPIsare showed in Table8:

Table 8. Detailed calculation of KPIs

KPI	Influential factors	IF weight	IF value	KPI amount
QPI	Rework/defects	[0.4091, 0.3846]	[0.4444, 0.5556]	[0.5043, 0.6931]
	Strategic quality management	[0.1818, 0.1923]	[0.5556, 0.6667]	
	Personnel quality training	[0.1818, 0.1923]	[0.4444, 0.5556]	
	Nonconformance rate	[0.2273, 0.2308]	[0.6667, 1]	
SSPI	Conflicts/disputes/claims	[0.6, 0.5263]	[0.8333, 1]	[0.6409, 0.9538]
	Change orders	[0.0667, 0.1579]	[0.6667, 0.8333]	
	Stakeholder satisfaction	[0.3333, 0.3158]	[0.5, 0.6667]	

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SFPI	Accident frequency ratio	[0.4, 0.45]	[0.6667, 1]	[0.4278, 0.7241]
	Safety equipment	[0.2667, 0.25]	[0.1111, 0.3333]	
	Safety training	[0.3333, 0.3	[0.4444, 0.5556]	
EPI	Inflation	[0.3913, 0.3571]	[0.5556, 0.6667]	[0.5173, 0.7013]
	Energy	[0.1304, 0.1429]	[0.4444, 0.5556]	
	Financial	[0.1739, 0.1786]	[0.4444, 0.5556]	
	Cash flow	[0.1304, 0.1429]	[0.6667, 1]	
	Developing cost	[0.1739, 0.1786]	[0.5556, 0.6667]	
MPI	Productivity	[0.2353, 0.25]	[0.4444, 0.5556]	[0.5686, 0.835]
	Errors	[0.5294, 0.5]	[0.6667, 1]	
	Benefits to Costs	[0.2353, 0.25]	[0.5556, 0.6667]	
PPI	Environmental condition	[0.1364, 0.1429]	[0, 0.1111]	[0.469, 0.7403]
	Accidental (unscheduled)	[0.1818, 0.1786]	[0.4444, 0.5556]	
	Government actions	[0.2727, 0.3214]	[0.5556, 0.6667]	
	Sanctions	[0.4091, 0.3571]	[0.6667, 1]	
CSPI	Equipment	[0.3913, 0.3448]	[0.4444, 0.5556]	[0.4831, 0.7623]
	Consume resources	[0.1304, 0.1379]	[0.6667, 1]	
	Capital Costs	[0.2174, 0.2069]	[0.3333, 0.4444]	
	Manpower	[0.2609, 0.3103]	[0.6667, 1]	

On the basis ofsection 3.1,the $\otimes CRPI$and $\otimes TRPI$valuesare derived by summing thenormalized individual weight of a KPI multiplied with its value. It should be noted that the weights of KPIs in formation of the $\otimes TRPI$ formulation might be varied with the weights of KPIs in constitution of $\otimes CRPI$.

Table 9. Calculation of $\otimes \boldsymbol{CRPI}$ with its elements

KPI	Weight($\otimes w_i$)	KPI amount	CRPI amount
EPI	[0.2899, 0.2920]	[0.5172, 0.7013]	[0.4964, 0.7691]
MPI	[0.1913, 0.2108]	[0.5686, 0.8349]	
PPI	[0.2460, 0.2545]	[0.4689, 0.7402]	
CsPI	[0.2530, 0.2620]	[0.4830, 0.7622]	

Table 1. Calculation of TRPI with its elements

KPI	Weight ($\otimes z_k$)	KPI amount	TRPI amount
EPI	[0.2765, 0.2884]	[0.5172, 0.7013]	[0.488, 0.791]
MPI	[0.2167, 0.2553]	[0.5686, 0.8349]	
PPI	[0.1808, 0.2120]	[0.4689, 0.7402]	
CsPI	[0.2872, 0.2827]	[0.4830, 0.7622]	

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Then,according to W-TODIM method,the weights of the CPI, SPI, QPI, SSPI, SFPI, and CRPI indicators are shown in Table 11 to influence on $\otimes\ GIEAC_{(\$)}$and $\otimes\ GIEAC_{(t)}$ formulae:

Table 2. Ultimate weights of KPIs in $\otimes\ GIEAC_{(\$)}$and $\otimes\ GIEAC_{(\$)}$formulae

WK_{kpi}	W-TODIM Weights	Z_{kpi}	W-TODIM Weights
WK_1	0.197466	Z_1	0.1572373
WK_2	0.161020	Z_2	0.1988687
WK_3	0.177892	Z_3	0.1890037
WK_4	0.162255	Z_4	0.1525147
WK_5	0.152602	Z_5	0.1479951
WK_6	0.148763	Z_6	0.1543801

Now, computing the $\otimes\ GIEAC_{(\$)}$and $\otimes\ GIEAC_{(\$)}$ are feasible as bellows:

$$\otimes\ GIEAC_{(\$)} = [60439.82\ .\ 80748.51] \tag{39}$$

$$\otimes\ GIEAC_{(t)} = [17.57351\ .\ 23.3106] \tag{40}$$

5. Discussion of results

According to the responses obtained in the previous section, it is important that those responses be interprete the results. From Eq.(52) which $\otimes\ GIEAC_{(\$)}$ is in range of 60439,82 to 80748,51 and in according to the ranking methodCo$\left(\otimes\ GIEAC_{(\$)}.\otimes\ A^{*}\right)$ =206221444.8 is higher than $\mathrm{Co}(\mathrm{BAC}\ .\otimes\ \mathrm{A}^{*})$=59839830.82.So, the project would be completed with more cost than the BAC. For time analysis,the range of $\otimes\ GIEAC_{(t)}$ is between 17,5735 and 23,3106. It is obtained that $\mathrm{Co}\left(\otimes\ GIEAC_{(t)}.\otimes\ A^{*}\right)$is equal to16.4571 and $\mathrm{Co}(\mathrm{PD}.\otimes\ A^{*})$is equal to6.3849. Thus, for time analysis could be stated that the project would be completed with more time than the PD.

As stated earlier, one of the parameters that in turn affects $\otimes\ GIEAC_{(\$)}$ and $\otimes\ GIEAC_{(t)}$isgreyprogresspercents of each activities.In this regard, a sensitivity analysis is accomplishedto determine the effects of different grey progress percentsusing montecarlo simulation.Thus, the progress percent of any activity is modified and the main concentration is the estimations of results. A total of 8,000 iterations are run. The related simulation results are demonstrated in Table12, Figs.1 and 2.

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Table 3. Summary of simulation results

$\otimes\ GIEAC_{(\$)}$	Maximum	Minimum	Mean	Standard deviation
Lower band	66226.3314	61988.4166	63994.5374	701.1737
Upper band	111763.2527	84688.4856	97092.6226	4837.3982
$\otimes\ GIEAC_{(t)}$	Maximum	Minimum	Mean	Standard deviation
Lower band	19.1228	17.9528	18.5349	0.1908
Upper band	32.4381	24.6331	28.3196	1.4810

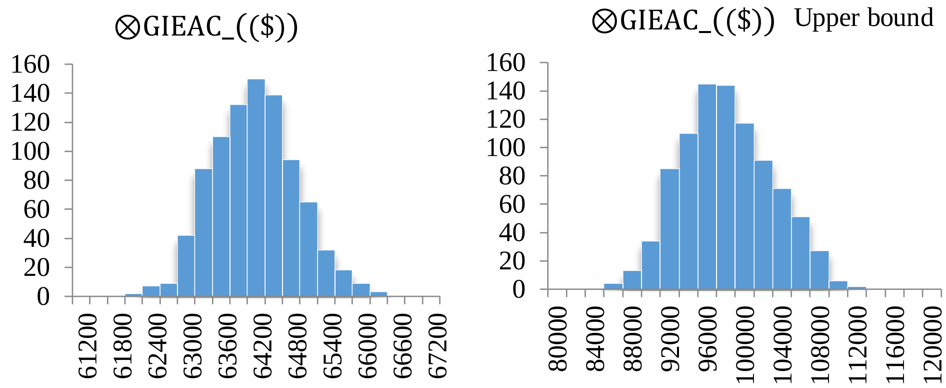

Figure 1. The$\otimes$ $GIEAC_{(\$)}$simulation

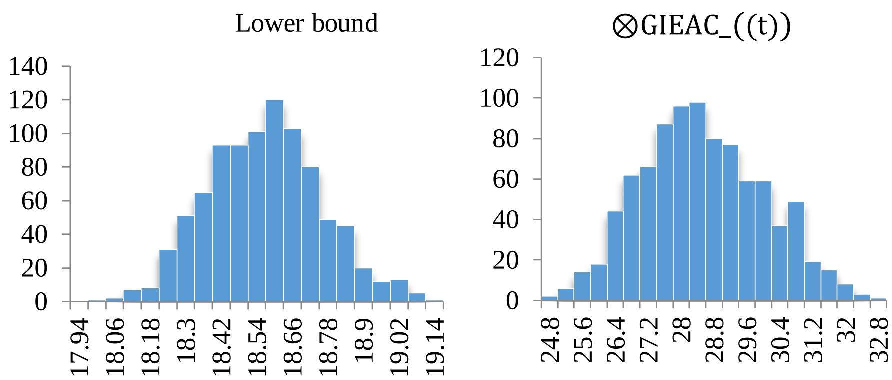

Figure 2.The$\otimes$ $GIEAC_{(t)}$simulation

6. Conclusion

Nature of projects is gradually changing to more sophisticated and dynamic manner, particularly in construction projects. Several approaches that consider uncertainty in many ways have been provided significant for the construction

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project to improve their performance and secure project success.This paperapplied the grey theoryto handle the uncertainty for itsprecision and simplicity in modeling. In the first part, a new grey-based EV analysis was presented to show the flexibility in calculations.In second part, several KPIs had beenassessed to focus on various aspects of a construction project whereas other past versions were merely concentrated on four KPIs to formulating the cost and time estimations at completion. Moreover, newW-TODIM method was applied to assign the weights of six KPIs in estimations formulae in the grey environment. The practitioner can comfortably understand the status of project by successful implementation of the approach in a construction environment. The results show that the integration of key performance indicators leads to better analysis, as well as more accurate prediction and presentation of information. Hence, project managers can monitor the status condition of critical ingredient comprehensively and make informed conscious decisions by maintaining the simplicity of analyzes.To depicts the model's applicability, a case study for construction project from the literature was selected and solved precisely. In order to expand the study and as future research, fuzzy clustering methods of the factors influencing project risks can be used to determine the earned value.

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